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APPROXIMATION OF POINT-SET IMAGES BY PLANE CURVES.(U)
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APPROXIMATION OF POINT-SET IMAGES BY PLANE CURVES

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ABSTRACT

A transform method is presented for detecting elements of a parametric family of curves in noisy point-set images. Local maxima of the transform in the parameter space will correspond to best approximations to parts of the data. The cases of linear, circular, and parabolic approximations are discussed.

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§1. Introduction

Statistical geometry is the study of how to restore pure geometric objects when we can only observe deformed versions of them. The case with which this paper is concerned is when the pure geometric objects are finite subsets of curve segments from a given parametric family. If we call the pure image I, then D(I), the deformed version, consists of a (random) point-subset of I with additive noise superimposed, with perhaps an additional point-process as background noise.

In this paper we shall only present an approximation method, similar to the Hough transform as described in [2] and to the chamfer matching in [1]. Additional Hough transform references are in [4]. Distributional considerations will be examined later. The general problem, which covers a wide range of distributional assumptions, appears below.

We suppose $\Omega \subset \underline{R}^d$ is some parameter space with $F = \{\{(x,y): f(\xi, x,y) = 0,\ (x,y) \in \underline{R}^2\}: \xi \in \Omega\}$ a family of curves in $\underline{R}^2$ parameterized by ξ. Let $(x_1,y_1),\ldots,(x_N,y_N)$ be a set of N points of $\underline{R}^2$ that arise as a deformed version of a set of segments of M curves parameterized by $\xi_1,\ldots,\xi_M$ of F. We shall assume that $(x_1,y_1),\ldots,(x_N,y_N)$ arises as a deformation of a point-process realization of the segments, plus additive noise.

Let ξ denote a curve of family F (we identify a curve with its parameter). A <u>segment</u> I of ξ of length L is a regular curve segment [3] of ξ parameterized by arc-length. That is, there is a function α: $[0,L] \to \underline{R}^2$ satisfying

(1) $f(\xi, \alpha_1(s), \alpha_2(s))=0 \qquad s\in[0,1]$

(2) α is one-to-one, except perhaps at the end points.

(3) The following identity holds:

$$\sigma = \int_0^\sigma \left|\frac{d\alpha}{ds}(s)\right| ds \qquad 0 \le \sigma \le 1.$$

Let $P = \{s_1<s_2<\ldots<s_N\}$ be a point-process realization of the interval $[0,L]$ consisting of N points, with an interpoint distance Δ that is a random variable with a known distribution. Let $e_1,\ldots,e_N$ be independent noise random variables with values in $\underline{R}^2$, each component having mean 0 and variance σ^2. Then

$$D(I)=\{(x_i,y_i)=(\alpha_1(s_i),\alpha_2(s_i))+e_i \quad : \quad i=1,\ldots,N\}$$

is the randomly deformed version of the image I. The above construction is extended to a set of M segments from F. In the rest of this paper we assume no background noise.

In Section 2 we discuss the case of fitting a single curve to a data set based on simple least-squares minimization. Although this direct method of fitting a single curve can be extended to fitting several curves simultaneously, difficulties are presented in that the number of curves present in an image is usually unknown, and that the optimization solution is complicated. In Section 3 we define a more robust procedure than least-squares fitting in which the parameter space is searched for local maxima after a transform method similar to the Hough transform. Finally the computations needed to carry out the procedure for linear, parabolic, and circular segments are given. We shall not consider in this paper methods for searching for local extrema.

§2. Fitting a single curve segment

For fitting a single curve segment of ξ to a set of data points $\{(x_1,y_1),\dots,(x_N,y_N)\}$ we can consider the least-squares minimization problem

$$\min_{(\xi,(\alpha_1,\beta_1),\dots,(\alpha_N,\beta_N))} \sum_{i=1}^{N} [(x_i-\alpha_i)^2+(y_i-\beta_i)^2]$$

$$\text{with} \quad \xi \in \Omega$$

$$f(\xi,\alpha_i,\beta_i) = 0 \quad i=1,\dots,N.$$

This problem is solved by fixing ξ and finding the solution $(\alpha_i^*(\xi),\beta_i^*(\xi))$ that minimizes $(x_i-\alpha_i)^2+(y_i-\beta_i)^2$ with $f(\xi,\alpha_i,\beta_i) = 0 \quad i=1,\dots,N$. By the Lagrange Multiplier Method we get the "normal equations" for $\alpha_i^*(\xi),\beta_i^*(\xi)$:

$$\begin{cases} \alpha_i-x_i+\lambda f_\alpha(\xi,\alpha_i,\beta_i) = 0 \\ \beta_i-y_i+\lambda f_\beta(\xi,\alpha_i,\beta_i) = 0 \\ f(\xi,\alpha_i,\beta_i) = 0 \end{cases}$$

The above minimization problem is now reduced to

$$\min_{\xi\in\Omega} \sum_{i=1}^{N} (x_i-\alpha_i^*(\xi))^2+(y_i-\beta_i^*(\xi))^2$$

a search for the minimum in parameter space.

In some cases solution of the normal equations is difficult, and the linear approximation to the constraints of the minimization problem is valid. We consider the linear approximation

$$f(\xi,x_i+\delta_i,y_i+\varepsilon_i) \cong$$

$$f(\xi,x_i,y_i)+\nabla_{\alpha,\beta} f(\xi,x,y)\binom{\delta_i}{\varepsilon_i}$$

$$i=1,\dots,N.$$

If $\alpha_i^* = x_i+\delta_i$, $\beta_i^* = y_i+\varepsilon_i$ are within the range of the above approximation, then the above problem may be approximated by

$$\min_{(\xi,(\delta_1,\varepsilon_1),\ldots,(\delta_N,\varepsilon_N))} \sum_{i=1}^{N} (\delta_i^{2}+\varepsilon_i^{2})$$

with $\xi\in\Omega$

$$\nabla_{\alpha,\beta}\, f(\xi,x_i,y_i)\binom{\delta_i}{\varepsilon_i} = -\, f(\xi,x_i,y_i) \quad i=1,\ldots,N.$$

For a fixed $\xi\in\Omega$ and $i=1,\ldots,N$ we define

$$v_i^*(\xi) = \min_{(\delta_i,\varepsilon_i)} (\delta_i^{2}+\varepsilon_i^{2})$$

with $f_\alpha(\xi,x_i,y_i)\delta_i+f_\beta(\xi,x_i,y_i)\varepsilon_i = -\, f(\xi,x_i,y_i)$.

It can be shown that

$$v_i^*(\xi) = \begin{cases} \dfrac{f(\xi,x_i,y_i)^2}{\|\nabla_{\alpha,\beta}f(\xi,x_i,y_i)\|^2} & \text{if } \nabla_{\alpha,\beta}f(\xi,x_i,y_i) \neq 0 \\ 0 \text{ otherwise} \end{cases}$$

The linearized least-squares problem then becomes

$$\min_{\xi\in\Omega} \sum_{i=1}^{N} v_i^*(\xi).$$

The above methods can be used in the case of approximating a set of M curves $\xi_1,\ldots,\xi_M$ to the data. We could make the same calculations as above but with $g(\xi_1,\ldots,\xi_M,x,y) = \prod_{j=1}^{M} f(\xi_i,x,y)$ in place of $f(\xi,x,y)$. We would then have a minimization problem in Ω^M, which is a rather unpractical situation. In the next section we discuss a transform method which results in searching for local extrema, but only in Ω.

§3. Fitting several curve segments simultaneously

In this section we consider a transform from the data set to the parameter space Ω in which search for local extrema allows one to extract the best fitting curve segments from F. As examples of the computation involved, we consider three cases: linear, parabolic, and circular segments. Note that multiple segments from the same curve are allowed. Attention is paid to the variance σ^2 of the noise as well as the distribution of Δ, the interpoint distance in a given segment.

Let $g(r)$ be a decreasing function on $r \geq 0$ satisfying

(1) $g(r)=0 \quad r \geq c_g$ for some $c_g > 0$

(2) smoothness assumptions on g as may be required.

A function g satisfying these requirements will be called an influence function. [In practice we choose c_g based on the variance σ^2 of the noise; for example

$$P(|X| < c_g) = .75$$

where X is a $N(0,\sigma^2)$ r.v. The influence function could take the form $g(r) = s(c_g^2 - r^2)$ for $0 \leq r \leq c_g$, and some $s>0$].

We consider the following problem:

$$\max_{(\xi,(\alpha_1,\beta_1),\ldots,(\alpha_N,\beta_N))} \sum_{i=1}^{N} g\left(\sqrt{(x_i-\alpha_i)^2+(y_i-\beta_i)^2}\right)$$

with $\quad \xi \in \Omega$

$$f(\xi,\alpha_i,\beta_i) = 0 \qquad i=1,\ldots, N$$

We fix $\xi \epsilon \Omega$ and solve for each $i=1,\ldots,N$

$$v_i^*(\xi) = \min_{(\alpha_i,\beta_i)} \sqrt{(x_i-\alpha_i)^2+(y_i-\beta_i)^2}$$

with $f(\xi,\alpha_i,\beta_i) = 0$

The <u>response function</u> $R(\xi)$ is defined by

$$R(\xi) = \sum_{i=1}^{N} g(v_i^*(\xi))$$

and the <u>response set</u> $S(\xi)$ by

$$S(\xi) = \{(\alpha_i^*,\beta_i^*) \quad i=1,\ldots,N: \quad v_i^*(\xi) < c_g\}$$

We note that $g(v_i^*(\xi))$ is positive only if the distance from the data point (x_i,y_i) to the curve ξ is less than the threshold value c_g as defined in the influence function. We can then expect $R(\xi)$ to be maximal (locally) at the ξ's that fit the largest proportion of the data set $(x_1,y_1),\ldots,(x_N,y_N)$. The response sets $S(\xi)$ at the maximal ξ's give the points of the curve ξ that best approximate the image before the additive noise. Recalling that we assumed the (a priori) knowledge of the interpoint distance Δ we can use this as a basis for clustering the response set $S(\xi)$ into segments.

In the next section we examine three cases of paramerized families of curves: linear, parabolic, and circular.

§4. Examples

A. Line segments

As in [2] the parameter space is (φ,γ), $\varphi\in[-\pi,\pi)$, $\gamma\in(-\infty,\infty)$, where a line is defined by the equation

$$\cos(\varphi)x + \sin(\varphi)y - \gamma = 0.$$

For each data point (x_i,y_i) we thus consider the problem

$$\begin{cases} \min[(\alpha-x_i)^2 + (\beta-y_i)^2] \\ \text{with } \cos(\varphi)\alpha + \sin(\varphi)\beta - \gamma = 0. \end{cases}$$

By means of the Lagrange Multiplier Method we can show that

$$\begin{cases} \alpha_i^* = \sin^2(\varphi)x_i - \sin(\varphi)\cos(\varphi)y_i + \gamma\cos(\varphi) \\ \beta_i^* = \cos^2(\varphi)y_i - \sin(\varphi)\cos(\varphi)x_i + \gamma\cos(\varphi) \\ v_i^* = |x_i\cos(\varphi) + y_i\sin(\varphi) - \gamma| \end{cases}$$

In practice, the parameter is discretized. For each data point (x_i,y_i) and each φ from the discretized set we compute $g(|x_i\cos(\varphi) + y_i\sin(\varphi) - \gamma|)$ for all γ such that $|x_i\cos(\varphi) + y_i\sin(\varphi) - \gamma| < c_g$. (thus we don't have to compute a value for each (φ,γ)). Summing all these contributions from all the data points gives the response function $R(\xi)$. After searching the parameter space for the locally maximal ξ, we compute the response set $S(\xi)$. Let $(\alpha,\beta)\in S(\xi)$. The transformation

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \to \begin{pmatrix}\cos(-\varphi) & -\sin(-\varphi)\\ \sin(-\varphi) & \cos(-\varphi)\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} - \begin{pmatrix}\gamma\\ 0\end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \cos(\varphi)\beta - \sin(\varphi)\alpha \end{pmatrix}$$

is used to transform the response set $S(\xi)$ to the y-axis.

A clustering procedure on $\underline{R}$ may then be used to find the segments of ξ.

B. Circular segments

We consider the parameter space with elements (φ,γ,ρ) $\varphi\in[-\pi,\pi)$, $\gamma>0$, $\rho> c_g$ specifying the curve

$$(\cos(\varphi)x + \sin(\varphi)y - \gamma)^2 + (\cos(\varphi)y - \sin(\varphi)x)^2 - \rho = 0$$

(the equation of the curve with center at polar coordinates (φ,γ) and radius ρ). For each data point (x_i,y_i) we then consider the problem

$$\min[(\alpha-x_i)^2 + (\beta-y_i)^2]$$

$$\text{with } (\cos(\varphi)\alpha + \sin(\varphi)\beta - \gamma)^2 + (\cos(\varphi)\beta - \sin(\varphi)\alpha)^2 - \rho = 0$$

It is easy to see (geometrically) that

$$\alpha_i{}^* = \frac{(x_i-\gamma\cos(\varphi))\rho}{\sqrt{(x_i-\gamma\cos(\varphi))^2+(y_i-\gamma\sin(\varphi))^2}} + \gamma\cos(\varphi)$$

$$\beta_i{}^* = \frac{(y_i-\gamma\sin(\varphi))\rho}{\sqrt{(x_i-\gamma\cos(\varphi))^2+(y_i-\gamma\sin(\varphi))^2}} + \gamma\sin(\varphi)$$

$$v_i{}^* = \left|\sqrt{(x_i-\gamma\sin(\varphi))^2+(y_i-\gamma\sin(\varphi))^2} - \rho\right|$$

Points (x_i,y_i) with $v_i{}^* < c_g$ give a positive contribution to the response function $R(\varphi,\gamma,\rho)$ and thus

$$R(\varphi,\gamma,\rho) = \sum_{i=1}^{N} v_i{}^*(\varphi,\gamma,\rho)$$

can be computed over the parameter space by fixing (φ,γ) and computing $g(v_i{}^*(\varphi,\gamma,\rho))$ for which ρ satisfies $v_i{}^*(\varphi,\gamma,\rho) < c_g$, $\rho>c_g$. As in the case of linear segments, after the local maxima of $R(\varphi,\gamma,\rho)$ have been located, the response sets

$S(\varphi,\gamma,\rho)$ are computed for those particular values, and a similar clustering (involving a decision procedure based on the distribution of Δ, the interpoint distance) can be implemented to extract segments.

C. Parabolic segments

The parameter space has elements (φ,γ,σ) $\varphi\in[-\pi,\pi)$, $\gamma>0$, $\sigma>0$ where a parabola is defined by

$$[\cos(\varphi)x + \sin(\varphi)y]^2 + \sin(\varphi)x - \cos(\varphi)y + \gamma = 0,$$

which is simply the fundamental parabola

$$x^2-y = 0$$

scaled by σ, increased by γ and rotated by φ. Thus for a data point (x_i,y_i) we first transform

$$x_i' \leftarrow \cos(\varphi)x_i + \sin(\varphi)y_i$$

$$y_i' \leftarrow -\sin(\varphi)x_i + \cos(\varphi)y_i - \gamma$$

and consider the problem

$$\min(\alpha-x_i')^2 + (\beta-y_i')^2$$
$$\text{with } \beta=\sigma\alpha^2.$$

We form the Lagrangian

$$L = (\alpha-x_i')^2 + (\beta-y_i')^2 - \lambda(\beta-\sigma\alpha^2).$$

The minimization results in the simultaneous equations

$$(\alpha-x_i') + \lambda\sigma\alpha = 0$$
$$(\beta-y_i') - \lambda = 0$$
$$\beta = \sigma\alpha^2$$

Solving these we get

$$h(\alpha) = \alpha^3 + \frac{[1-2\sigma y_i']}{2\sigma}\alpha - \frac{x_i'}{2\sigma} = 0$$

Then $[\alpha_i^*]'$ will be a root of this equation, where ' denotes the above transformation. Correspondingly $[\beta_i^*]' = \sigma([\alpha_i^*]')^2$, $v_i^* = \sqrt{(x_i' - [\alpha_i^*]')^2 + (y_i' - [\beta_i^*]')^2}$ ($[\alpha_i^*]'$ will be the real root of $h(\alpha) = 0$ that minimizes v_i^*).

No more will be said about searching for the local maxima of $R(\varphi,\gamma,\sigma) = \sum_{i=1}^{N} g(v_i^*(\varphi,\gamma,\sigma))$, but once the response set $\{(\alpha_i^*, \beta_i^*): i=1,\ldots,N\}$ corresponding to a particular parameter has been found, it will be necessary to perform a clustering procedure to extract segments. We first transform the response set to $\{([\alpha_i^*]', [\beta_i^*]')\}$ using the transformation given above, so the points lie on $\beta=\sigma\alpha^2$. By transforming the parabola $\beta=\sigma\alpha^2$ to the line $y=0$ (preserving arc length) we can use the same clustering procedure in the linear case. Such a transformation [3] is given by $(\alpha,\sigma\alpha^2) \to (s,0)$ where $s = \mathrm{sgn}(\alpha)(1/4\sigma)[2\sigma|\alpha|\sqrt{1+4\sigma^2\alpha^2} + \ln(2\sigma|\alpha|+\sqrt{1+4\sigma^2\alpha^2})]$.

5. Conclusion

We have presented in this paper a procedure for fitting curve segments to a data set when the curves originate from a given parametric family. The method involves a transform into the parameter space where the largest local maxima indicate the best fitting curves to the data. The data points within a threshold neighborhood of a given curve from the previous step are projected onto the curve. The resulting point set can then be clustered to extract curve segments. The parameter spaces and basic computations for carrying out this procedure have been done for linear, circular, and parabolic segments.

A survey of the many variants of the original Hough Transform can be found in [4]. Basically, in these techniques the additive noise (due to scatter) is taken care of in the quantization of the parameter space. As noted in [5] this has led to unsatisfactory results when random noise is not removed by the quantization procedure, and a maximum entropy quantization is introduced (in the case the noise distribution is known). In these methods, after transforming and thresholding, a clustering procedure is performed to extract the unknown curves. The method developed in this paper is not as sensitive to how the parameter space is quantized, and replaces clustering with the problem of detecting local maxima. Also included in [4] is a discussion of detection of circular and parabolic segments, and the relationship of the Hough Transform to template matching in the case where all the curves to be detected are translations of each other.

A generalized Hough transform is presented in [6], and its equivalence to a formulation of the point pattern matching problem is given. In this formulation a set of object points $O \subseteq \underline{R}^n$, a set of feature points $P \subseteq R^m$, and a set of parameterized functions $f_\xi : \underline{R}^n \to R^m$ are given, and the problem is to find f_ξ that maximizes the number of points in O that are mapped onto points in P by f_ξ. The GHT procedure is to consider all point pairs (o,p) $o \in O$, $p \in P$ and compute all ξ such that $f_\xi(o) = p$ (up to quantization level). In practice an array of accumulators HT (one for each of the quantized parameter values) is constructed, and HT(ξ) is incremented by unity, where ξ is an outcome of the above computation. In the modified GHT we would increment an accumulator ξ by

$$g(d(o, \{\bar{o}: f_\xi(\bar{o}) = p\}))$$

where d is the infimum of the Euclidean distances from o to all points $\bar{o}$ satisfying $f_\xi(\bar{o}) = p$, and g is an influence function as defined above.

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